Assignment 7.

1. It is given that $\int_1^a \ln(2x)\,\mathrm{d}x = 1$, where $a > 1$.

   Show that $a = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a}\right)$, where $\exp(x)$ denotes e^x. [6]

2. (a) Use the substitution $u = \tan x$ to show that, for $n \neq -1$, [4]

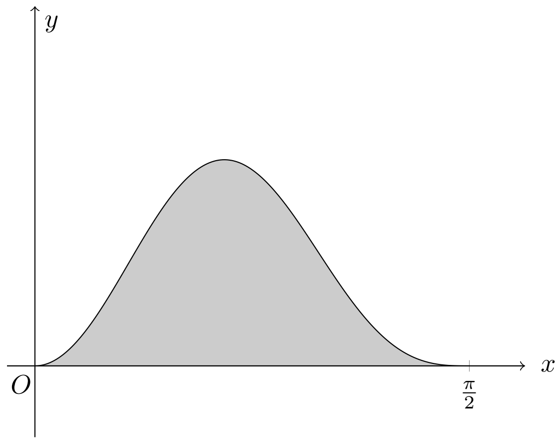
$$\int_0^{\frac{1}{4}\pi} \left(\tan^{n+2} x + \tan^n x\right)\,\mathrm{d}x = \frac{1}{n+1}.$$

   (b) Hence find the exact value of

   i. $\int_0^{\frac{1}{4}\pi} \left(\sec^4 x - \sec^2 x\right)\,\mathrm{d}x$, [3]

   ii. $\int_0^{\frac{1}{4}\pi} \left(\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x\right)\,\mathrm{d}x$. [3]

3. The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.



(a) Find the $x-$coordinate of M. [6]

(b) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the $x-$axis. [4]

4. (†) Evaluate the integral $\displaystyle\int_{-1}^{1} \left| \frac{x}{x+2} \right| \mathrm{d}x$ [7]

Total mark of this assignment: $26 + 7$.
The symbol (†) indicates a bonus question. Finish other questions before working on this one.